

# Performance Analysis of Cooperative-based Multi-hop Transmission Protocols in Underlay Cognitive Radio with Hardware Impairment

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## Abstract

In this paper, we study performances of multi-hop transmission protocols in underlay cognitive radio (CR) networks under impact of transceiver hardware impairment. In the considered protocols, cooperative communication is used to enhance reliability of data transmission at each hop on an established route between a secondary source and a secondary destination. For performance evaluation, we derive exact and asymptotic closed-form expressions of outage probability and average number of time slots over Rayleigh fading channel. Then, computer simulations are performed to verify the derivations. Results present that the cooperative-based multi-hop transmission protocols can obtain better performance and diversity gain, as compared with multi-hop scheme using direct transmission (DT). However, with the same number of hops, these protocols use more time slots than the DT protocol.

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## 1. Introduction

In wireless networks such as adhoc networks [1] and wireless sensor networks [2], multi-hop relaying scenarios are used widely due to far distances between source node and destination node. In conventional multi-hop scheme, the direct transmission (DT) is used to relay the source's data to the destination [3, 4]. Although the implementation of the DT protocol is easy in practice, its performance significantly degrades in fading environments [4]. To enhance performances for the multi-hop schemes, in published literature such as [5, 6], the authors proposed multi-hop diversity relaying protocols in which a relay is selected to cooperate with the transmitter at each hop to forward the data to next hop. In [7], a

cluster-based cooperative protocol for multi-hop transmission was proposed and analyzed. In this protocol, the cluster node with the maximum instantaneous channel gain will serve as the sender for the next cluster. In [8, 9, 10, 11, 12], the authors proposed cooperative routing protocols in which intermediate nodes on the established route exploit the cooperative communication to forward the source data. Although performances of these protocols significantly are enhanced, as compared with the DT protocol, their implementation which requires a high synchronization between all the intermediate nodes, is a very difficult work.

Recently, multi-hop relaying protocols in cognitive radio (CR) networks have gained much attention as an efficient method to enhance the coverage and channel capacity for secondary

networks. Different with the conventional wireless networks, transmit powers of secondary users are limited by interference thresholds given by primary users (PU) [13, 14]. Due to the limited power, performances of multi-hop CR protocols significantly degrades [15, 16], especially in CR schemes with multiple PUs [17]. Again, cooperative communication protocols are employed to enhance quality of service (QoS) for the secondary networks. In [18, 19], underlay cooperative routing protocols with and without using combining techniques were proposed and analyzed, respectively. Results in [18, 19] presented that the proposed schemes provide an impressive performance gain as compared with the DT model.

So far, almost published works related to the multi-hop networks assumed that the transceivers are perfect. However, in practice, they are suffered from impairments due to I/Q imbalance, high power amplifier non-linearities and phase noise [20]. Due to the hardware noises, the channel capacity obtained at high signal-to-noise ratio (SNR) region is limited [21]. In [22, 23], the authors considered two-way relaying protocols under the presence of the hardware impairments over Rayleigh fading channel and Nakagami- $m$  fading channel, respectively. Works in [24] and [25] proposed relay selection methods to obtain diversity order as well as compensate the performance loss due to the hardware impairment. To the best of our knowledge, the most related to our work is the cognitive decode-and-forward relaying protocol proposed in [26]. However, the authors in [26] only considered the dual-hop network with selection combining technique at the destination. Moreover, only outage probability of the proposed scheme was evaluated in [26], while other important metrics such as diversity gain and spectrum efficiency were not considered. In this paper, we study performances of cooperative-based multi-hop protocols in underlay CR networks under the impact of the hardware impairment. The main contributions of this paper can be summarized as follows:

- We propose two multi-hop protocols in which either conventional cooperative (CC) protocol or incremental cooperative (IR) protocol [27] is used to enhance quality of the data transmission at each hop. In the CC protocol, the receiver at each hop is equipped with maximal ratio combining (MRC) technique to combine the received data [27]. In the IR protocol, the relay link is only used if the quality of the direct link is poor [27].
- We derive exact closed-form expressions of outage probability for the proposed schemes over Rayleigh fading channels. Moreover, we also derive an exact expression of average number of the time slots for the IR protocol. Then, Monte-Carlo simulations are presented to verify our derivations.
- To provide more insights into the system performance, we also derive the asymptotic outage probability where both diversity and coding gains are obtained.
- Finally, we compare the performance of the proposed protocols with the DT protocol to show the advantages of our schemes.

The rest of this paper is organized as follows. The system model of the proposed protocols is described in Section II. In Section III, the expressions of the outage probability and the average number of time slots are derived. The simulation results are shown in Section III. Finally, this paper is concluded in Section V.

## 2. System Model

Figure 1 illustrates the system model of the proposed cooperative-aided multi-hop transmission protocols in underlay cognitive radio. In this figure, the secondary source  $T_0$  transmits its data to the secondary destination  $T_M$  via a multi-hop model. We assume that an  $M$ -hop route between the secondary source and the secondary destination (with  $M - 1$  intermediate nodes, i.e.,  $T_1, T_2, \dots, T_{M-1}$ ) is established by

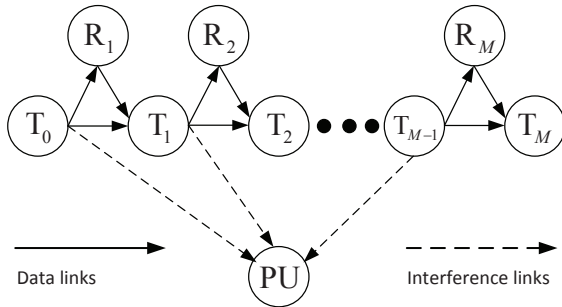


Fig. 1: Cooperative-aided multi-hop transmission protocol in underlay cognitive radio.

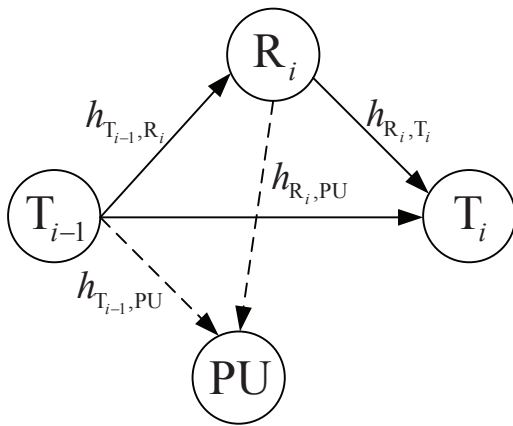


Fig. 2: Cooperative communication at the  $i$ th hop.

some methods on network layer such as Adhoc On-demand Distance Vector (AODV) [28]. At each hop on the routing path, a secondary relay is used to help the communication at that hop. We denote  $R_i$  as the relay of the  $i$ th hop,  $i \in \{1, 2, \dots, M\}$ . In underlay cognitive radio, the transmit power of all secondary transmitters must satisfy the interference threshold given at the primary user (PU) [29]<sup>1</sup>.

We assume that all of the nodes are equipped with only a antenna and operate on half-duplex mode. Next, we consider the data transmission

at the  $i$ th hop with three different techniques (see Fig. 2), i.e., conventional cooperation (CC), incremental cooperation (IR) and direct transmission (DT).

**In the CC protocol**, the data transmission at the  $i$ th hop is split into two time slots. At the first time slot, node  $T_{i-1}$ , which is assumed to receive the source data successfully before, transmits the source data to node  $T_i$  and relay  $R_i$ . At the end of the first time slot, relay  $R_i$  attempts to decode the received data. If the decoding at this node is successful, it forwards the decoded data to  $T_i$  at the second time slot. Then, node  $T_i$  combines the data received from  $T_{i-1}$  and  $R_i$  by using MRC technique. If the relay  $R_i$  cannot receive the source data successfully, it will not retransmit the data to  $T_i$ , and in this case, node  $T_i$  will decode the source data from the data received from  $T_{i-1}$ .

**In the IR protocol**, node  $T_{i-1}$  also broadcasts the source data to  $T_i$  and  $R_i$  at the first time slot. Then, nodes  $T_i$  and  $R_i$  try to decode the received data. If  $T_i$  can decode the data correctly, it sends back an ACK message to  $T_{i-1}$  and  $R_i$  to inform the decoding status. In this case, the data transmission at this hop is successful and hence the relay  $R_i$  does nothing. If the decoding at  $T_i$  is unsuccessful, it generates a NACK message to request a retransmission from  $R_i$ . The relay  $R_i$  then uses the second time slot to forward the source data to  $T_i$  if this node can decode the source data successfully. In this case, node  $T_i$  again attempts to decode the source data. If it fails again, the data is dropped at this hop. The advantage of the IR protocol, as compared with the CC protocol, is that when the quality of the  $T_{i-1} \rightarrow T_i$  link is good, the IR only uses one time slot to transmit the data, which enhances the spectrum efficiency. Moreover, in the IR protocol, the receiver  $T_i$  does not use any combining techniques to combine the received data, which reduces the complexity of the decoding process at this node.

**In the DT protocol**, node  $T_{i-1}$  directly transmits the source data to node  $T_i$ . In this scheme, if  $T_i$  cannot decode the data successfully, the data is dropped at this hop. We can observe that the DT protocol only uses one time slot at

<sup>1</sup>In Fig. 1, for ease of presentation, we would not show the interference links between the secondary relays and the primary user.

each hop. However, the data transmission at each hop of this protocol is less reliable than that of the CC and IR protocols.

Hereafter, we denote CC (or IR or DT) as the multi-hop transmission scheme in which the CC (or IR or DT) technique is used to transmit the data at each hop. We also assume that the density of secondary users in secondary network is high enough so that each hop on the routing path can select a secondary relay for the cooperation.

### 3. Performance Evaluation

#### 3.1. Channel model

Let us denote  $h_{X,Y}$  as the channel coefficient between nodes  $X$  and  $Y$ , where  $X, Y \in \{T_{i-1}, R_i, T_i, PU\}$  and  $i \in \{1, 2, \dots, M\}$ . Assume that  $h_{X,Y}$  follows Rayleigh distribution, hence, channel gain  $\gamma_{X,Y}$ , i.e.,  $\gamma_{X,Y} = |h_{X,Y}|^2$ , is an exponential random variable (RV). As presented in [6, eq. (1)], the cumulative density function (CDF) and the probability density function (PDF) of  $\gamma_{X,Y}$  can be given, respectively, as

$$F_{\gamma_{X,Y}}(z) = 1 - \exp(-\lambda_{X,Y}z), \quad (1)$$

$$f_{\gamma_{X,Y}}(z) = \lambda_{X,Y} \exp(-\lambda_{X,Y}z), \quad (2)$$

where  $\lambda_{X,Y} = d_{X,Y}^\beta$  with  $d_{X,Y}$  being the distance between  $X$  and  $Y$  and  $\beta$  being the path-loss exponent.

#### 3.2. Signal-to-noise and interference ratio (SNIR) formulation

Considering the communication between the transmitter  $X$  and the receiver  $Y$ ,  $X \in \{T_{i-1}, R_i\}$ ,  $Y \in \{T_i, R_i\}$ , the data received at  $Y$  can be expressed by

$$r_Y = \sqrt{P_X} h_{X,Y} (x + \eta_X^t) + \eta_Y^r + g_Y, \quad (3)$$

where  $P_X$  is transmit power of  $X$ ,  $x$  is the source data,  $\eta_X^t$  is hardware noise caused by the impairment in the transmitter  $X$ ,  $\eta_Y^r$  is noise from the hardware impairment in the receiver  $Y$  and  $g_Y$  is Gaussian noise at  $Y$ , which is modeled as Gaussian RV with zero-mean and variance  $\sigma^2$ .

Similar to [29, 30, 31], the transmit power  $P_X$  is limited by the interference threshold  $I_{th}$  at the PU as follows:

$$P_X = I_{th} / \gamma_{X,PU}, \quad (4)$$

Considering the hardware noises  $\eta_X^t$  and  $\eta_Y^r$ , they can be theoretically modeled as in [21]:

$$\eta_X^t \sim CN(0, \kappa_X^t P_X), \quad (5)$$

$$\eta_Y^r \sim CN(0, \kappa_Y^r P_X |h_{X,Y}|^2), \quad (6)$$

where  $CN(a, b)$  indicates circularly-symmetric complex Gaussian distributed variables in which  $a$  and  $b$  are mean and variances, respectively,  $\kappa_X^t$  and  $\kappa_Y^r$ ,  $\kappa_X^t, \kappa_Y^r \geq 0$ , characterize the level of hardware impairments in the transmitter  $X$  and receiver  $Y$ , respectively.

For ease of presentation and analysis, we assume that all of the nodes have the same structure so that their hardware impairment levels are same, i.e.,  $\kappa_X^t = \kappa_1$  and  $\kappa_Y^r = \kappa_2$ . However, if the hardware impairment levels are different, the obtained results in this paper are still used to derive the upper-bound and lower-bound expressions of the outage probability for the considered protocols.

From (3)-(5), the instantaneous signal-to-noise and interference ratio (SNIR) received at  $Y$  can be expressed as

$$\Psi_{X,Y} = \frac{\gamma_{X,Y} I_{th} / \gamma_{X,PU}}{(\kappa_1 + \kappa_2) \gamma_{X,Y} I_{th} / \gamma_{X,PU} + \sigma^2}. \quad (7)$$

By using (7), we can obtain the instantaneous SNIR of the  $T_{i-1} \rightarrow T_i$ ,  $T_{i-1} \rightarrow R_i$  and  $R_i \rightarrow T_i$  links, respectively as

$$\begin{aligned} \Psi_{T_{i-1}, T_i} &= \frac{Q \gamma_{T_{i-1}, T_i} / \gamma_{T_{i-1}, PU}}{\kappa Q \gamma_{T_{i-1}, T_i} / \gamma_{T_{i-1}, PU} + 1}, \\ \Psi_{T_{i-1}, R_i} &= \frac{Q \gamma_{T_{i-1}, R_i} / \gamma_{T_{i-1}, PU}}{\kappa Q \gamma_{T_{i-1}, R_i} / \gamma_{T_{i-1}, PU} + 1}, \\ \Psi_{R_i, T_i} &= \frac{Q \gamma_{R_i, T_i} / \gamma_{R_i, PU}}{\kappa Q \gamma_{R_i, T_i} / \gamma_{R_i, PU} + 1}, \end{aligned} \quad (8)$$

where  $Q = I_{th} / \sigma^2$  and  $\kappa = \kappa_X^t + \kappa_Y^r$ . Moreover, if MRC combiner is used, the SNIR received at  $T_i$  can be obtained as [29, eq. (8)]

$$\Psi_{MRC} = \Psi_{T_{i-1}, T_i} + \Psi_{R_i, T_i}. \quad (9)$$

### 3.3. Outage probability analysis

In this subsection, we derive exact and asymptotic expressions of outage probability for the considered protocols. Outage probability is defined as the probability that the received SNIR at a receiver is less than a predetermined threshold, i.e.,  $\gamma_{th}$ . With this definition, a receiver can be assumed to decode the data successfully if its received SNIR is above the threshold  $\gamma_{th}$ . Otherwise, this node cannot receive the data correctly.

#### 3.3.1. DT protocol

In this protocol, the outage probability at the  $i$ th hop can be given by

$$\text{Out}_i^{\text{DT}} = \Pr[\Psi_{T_{i-1}, T_i} < \gamma_{th}]. \quad (10)$$

Substituting  $\Psi_{T_{i-1}, T_i}$  in (8) into (10) yields

$$\text{Out}_i^{\text{DT}} = \begin{cases} 1; & \text{if } \kappa \geq 1/\gamma_{th} \\ \Pr\left[\frac{\gamma_{T_{i-1}, T_i}}{\gamma_{T_{i-1}, \text{PU}}} < \frac{\gamma_{th}}{(1-\kappa\gamma_{th})Q}\right]; & \text{if } \kappa < 1/\gamma_{th} \end{cases}. \quad (11)$$

We can observe from (11) that when the hardware impairment level  $\kappa$  is larger than  $1/\gamma_{th}$ , the communication between  $T_{i-1}$  and  $T_i$  is always in outage. For  $\kappa < 1/\gamma_{th}$ , the outage probability can be calculated by using [29, eq. (3)] as

$$\text{Out}_i^{\text{DT}} = \frac{\lambda_{T_{i-1}, T_i} \gamma_{th}}{\lambda_{T_{i-1}, T_i} \gamma_{th} + \lambda_{T_{i-1}, \text{PU}} (1 - \kappa \gamma_{th}) Q}. \quad (12)$$

Due to the independence of hops, the end-to-end outage probability of the DT protocol can be given, similarly as [5, eq. (15)]

$$P_{\text{out}}^{\text{DT}} = 1 - \prod_{i=1}^M (1 - \text{Out}_i^{\text{DT}}). \quad (13)$$

By substituting  $\text{Out}_i^{\text{DT}}$  in (12) into (13), we can obtain an exact closed-form expression of the outage probability for the DT protocol. It is obvious from (12) and (13) that the end-to-end outage probability increases with the increasing of  $\kappa$  and the decreasing of  $Q$ . To provide more insights into the outage performance, we next derive an asymptotic expression for  $P_{\text{out}}^{\text{DT}}$

at high  $Q$  value, i.e.,  $Q \rightarrow +\infty$ . Indeed, by using the approximation  $x/(1+x) \stackrel{x \rightarrow 0}{\approx} x$ , i.e.,  $x = \lambda_{T_{i-1}, T_i} \gamma_{th} / (\lambda_{T_{i-1}, \text{PU}} (1 - \kappa \gamma_{th}) Q)$ , for (12), we have

$$\text{Out}_i^{\text{DT}} \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{T_{i-1}, T_i}}{\lambda_{T_{i-1}, \text{PU}}} \frac{\gamma_{th}}{1 - \kappa \gamma_{th}} \frac{1}{Q}. \quad (14)$$

Then, an approximate expression of  $P_{\text{out}}^{\text{DT}}$  at high  $Q$  values can be given by

$$P_{\text{out}}^{\text{DT}} \stackrel{Q \rightarrow +\infty}{\approx} \sum_{i=1}^M \text{Out}_i^{\text{DT}} \approx \left( \sum_{i=1}^M \frac{\lambda_{T_{i-1}, T_i}}{\lambda_{T_{i-1}, \text{PU}}} \right) \frac{\gamma_{th}}{1 - \kappa \gamma_{th}} \frac{1}{Q}. \quad (15)$$

From (15), the diversity gain of the DT scheme can be easily determined as

$$\begin{aligned} \text{Div}_{\text{DT}} &= - \lim_{Q \rightarrow +\infty} \frac{\log(P_{\text{out}}^{\text{DT}})}{\log(Q)} \\ &= - \lim_{Q \rightarrow +\infty} \frac{\log\left(\left(\sum_{i=1}^M \frac{\lambda_{T_{i-1}, T_i}}{\lambda_{T_{i-1}, \text{PU}}}\right) \frac{\gamma_{th}}{1 - \kappa \gamma_{th}} \frac{1}{Q}\right)}{\log(Q)} \\ &= 1. \end{aligned} \quad (16)$$

As shown in (16), the DT scheme obtains the diversity order of 1 but its coding gain is reduced by an amount of  $G_{\text{DT}} = -10 \log_{10}(1 - \kappa \gamma_{th})$ , as compared with the corresponding scheme in which transceiver hardware is perfect.

#### 3.3.2. IR protocol

In this protocol, the outage probability at the  $i$ th hop can be formulated by

$$\begin{aligned} \text{Out}_i^{\text{IR}} &= \Pr[\Psi_{T_{i-1}, T_i} < \gamma_{th}, \Psi_{T_{i-1}, R_i} < \gamma_{th}] \\ &+ \Pr[\Psi_{T_{i-1}, T_i} < \gamma_{th}, \Psi_{T_{i-1}, R_i} \geq \gamma_{th}, \Psi_{R_i, T_i} < \gamma_{th}]. \end{aligned} \quad (17)$$

The first term in (17) presents probability that nodes  $R_i$  and  $T_i$  cannot decode the data correctly in the first time slot, while the second term indicates the event the relay  $R_i$  correctly receives the data but the decoding status at  $T_i$  at both time slots is unsuccessful.



$$\begin{aligned}
\text{Out}_i^{\text{IR}} = & 1 - \frac{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + \lambda_{T_{i-1},T_i}\gamma_{th}} - \frac{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + \lambda_{T_{i-1},R_i}\gamma_{th}} \\
& + \frac{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + (\lambda_{T_{i-1},T_i} + \lambda_{T_{i-1},R_i})\gamma_{th}} \\
& + \left[ \frac{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + \lambda_{T_{i-1},R_i}\gamma_{th}} - \frac{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + (\lambda_{T_{i-1},T_i} + \lambda_{T_{i-1},R_i})\gamma_{th}} \right] \\
& \times \frac{\lambda_{R_i,T_i}\gamma_{th}}{\lambda_{R_i,T_i}\gamma_{th} + \lambda_{R_i,\text{PU}}(1 - \kappa\gamma_{th})Q}. \tag{18}
\end{aligned}$$

**Proposition 1:** Under the presence of hardware impairment, if  $\kappa \geq 1/\gamma_{th}$  then  $\text{Out}_i^{\text{IR}} = 1$ , and if  $\kappa < 1/\gamma_{th}$ ,  $\text{Out}_i^{\text{IR}}$  can be expressed by an exact closed-form expression as in (18) at the top of next page.

*Proof*

With  $\kappa \geq 1/\gamma_{th}$ , we can easily obtain  $\text{Out}_i^{\text{IR}} = 1$ . For the case where  $\kappa < 1/\gamma_{th}$ , the proof is given in Appendix A.

Also, the end-to-end outage probability of the IR protocol can be expressed as

$$P_{\text{out}}^{\text{IR}} = 1 - \prod_{i=1}^M (1 - \text{Out}_i^{\text{IR}}). \tag{19}$$

In order to provide useful insights into the system performance such as diversity gain, we derive the asymptotic expression for  $P_{\text{out}}^{\text{IR}}$  at high  $Q$  values (see Corollary 1 below).

**Corollary 1:** When  $\kappa < 1/\gamma_{th}$ , the end-to-end outage probability  $P_{\text{out}}^{\text{IR}}$  can be approximated at high  $Q$  region by

$$\begin{aligned}
P_{\text{out}}^{\text{IR}} \stackrel{Q \rightarrow +\infty}{\approx} & \left[ \sum_{i=1}^M \frac{\lambda_{T_{i-1},T_i}}{\lambda_{T_{i-1},\text{PU}}} \left( \frac{2\lambda_{T_{i-1},R_i}}{\lambda_{T_{i-1},\text{PU}}} + \frac{\lambda_{R_i,T_i}}{\lambda_{R_i,\text{PU}}} \right) \right] \\
& \times \left( \frac{\gamma_{th}}{1 - \kappa\gamma_{th}} \right)^2 \frac{1}{Q^2}. \tag{20}
\end{aligned}$$

*Proof*

We proved this Corollary in Appendix B.

From the results in (20), it can be obtained that the IR protocol provides a diversity order of 2,

i.e.,

$$\begin{aligned}
\text{Div}_{\text{IR}} = & - \lim_{Q \rightarrow +\infty} \frac{\log(P_{\text{out}}^{\text{IR}})}{\log(Q)} \\
= & 2. \tag{21}
\end{aligned}$$

Moreover, we can see from (20) that due to the hardware impairment, the coding gain loss is  $G_{\text{IR}} = -20\log_{10}(1 - \kappa\gamma_{th})$ .

### 3.3.3. CC protocol

In this protocol, we can formulate the outage probability at the  $i$ th hop as follows:

$$\begin{aligned}
\text{Out}_i^{\text{CC}} = & \Pr[\Psi_{T_{i-1},R_i} < \gamma_{th}, \Psi_{T_{i-1},T_i} < \gamma_{th}] \\
& + \Pr[\Psi_{T_{i-1},R_i} \geq \gamma_{th}, \Psi_{\text{MRC}} < \gamma_{th}]. \tag{22}
\end{aligned}$$

In the RHS of the equation above, the first term takes the same form with that in (17), while the second term presents the probability that  $R_i$  can decode the data correctly but  $T_i$  cannot. Next, we will present the exact expression of  $\text{Out}_i^{\text{CC}}$  via Proposition 2.

**Proposition 2:** If  $\kappa \geq 1/\gamma_{th}$ , the outage probability  $\text{Out}_i^{\text{CC}}$  equals 1, otherwise, i.e.,  $\kappa < 1/\gamma_{th}$ , an exact closed-form expression of  $\text{Out}_i^{\text{CC}}$  can be given by (23) (see the top of next page), where  $a_0, a_1, a_2, b_1$  and  $b_2$  are given by (C.10) in Appendix C.

*Proof*

Also, we easily obtain that  $\text{Out}_i^{\text{IR}} = 1$  if  $\kappa \geq 1/\gamma_{th}$ . In the case that  $\kappa < 1/\gamma_{th}$ , we will present the proof in Appendix C.

$$\begin{aligned} \text{Out}_i^{\text{CC}} = & 1 - \frac{\lambda_{T_{i-1},\text{PU}}\gamma_{th}}{\lambda_{T_{i-1},\text{PU}}\gamma_{th} + \lambda_{T_{i-1},T_i}(1 - \kappa\gamma_{th})Q} - \frac{\lambda_{T_{i-1},\text{PU}}\gamma_{th}}{\lambda_{T_{i-1},\text{PU}}\gamma_{th} + \lambda_{T_{i-1},R_i}(1 - \kappa\gamma_{th})Q} \\ & + \frac{\lambda_{T_{i-1},\text{PU}}\gamma_{th}}{\lambda_{T_{i-1},\text{PU}}\gamma_{th} + (\lambda_{T_{i-1},T_i} + \lambda_{T_{i-1},R_i})(1 - \kappa\gamma_{th})Q} + a_0 \left( \frac{b_1\gamma_{th}}{a_1(a_1 - \gamma_{th})} + b_2 \log \left( \frac{a_1(a_2 - \gamma_{th})}{(a_1 - \gamma_{th})a_2} \right) \right). \end{aligned} \quad (23)$$

Similarly, an exact expression of the end-to-end outage probability for the CC protocol is given as

$$P_{\text{out}}^{\text{CC}} = 1 - \prod_{i=1}^M (1 - \text{Out}_i^{\text{CC}}). \quad (24)$$

Next, in Corollary 2 below, we derive asymptotic closed-form expression of  $P_{\text{out}}^{\text{CC}}$  at high  $Q$  regimes.

**Corollary 2:** When  $\kappa < 1/\gamma_{th}$ , the end-to-end outage probability  $P_{\text{out}}^{\text{IR}}$  can be approximated at high values of  $Q$  as in (25) at the top of next page.

*Proof*

Proof is presented in Appendix D.

Moreover, when  $\kappa = 0$ , (25) becomes

$$P_{\text{out}}^{\text{CC}} \stackrel{Q \rightarrow +\infty}{\approx} \sum_{i=1}^M \left[ \frac{2\lambda_{T_{i-1},R_i}\lambda_{T_{i-1},T_i}}{(\lambda_{T_{i-1},\text{PU}})^2} + \frac{\lambda_{T_{i-1},T_i}\lambda_{R_i,T_i}}{2\lambda_{T_{i-1},\text{PU}}\lambda_{R_i,\text{PU}}} \right] \left( \frac{\gamma_{th}}{Q} \right)^2. \quad (26)$$

From (25) and (26), it is obvious that the diversity gain of the CC protocol is 2, i.e.,  $\text{Div}_{\text{CC}} = 2$ .

### 3.4. Average number of time slots

In this subsection, we evaluate performance of the DT, IR and CC protocols via the metrics: average number of time slots used for a successful transmission between the source to the destination. It is obvious that the DT always uses  $M$  time slots to transmit the data, while the time slots used in the CC protocol is always  $2M$ .

Considering the successful data transmission in the IR protocol, we denote  $S_1$  as set of the hops that use only one time slot to transmit the data. It can be assumed that  $S_1 = \{j_1, j_2, \dots, j_L\}$ ,

where  $0 \leq L \leq M$  and  $j_1, j_2, \dots, j_L \in \{1, 2, \dots, M\}$ . Hence, if we denote  $S_2$  as the set of the hops using two time slots to transmit the data, then  $S_2 = \{j_{L+1}, j_{L+2}, \dots, j_M\}$  and  $S_1 \cup S_2 = \{1, 2, \dots, M\}$ . For example, if  $L = 0$ , then all of hops uses two time slots, i.e.,  $S_1 = \{\emptyset\}$  and  $S_2 = \{1, 2, \dots, M\}$ . For another example, if  $L = M$ , then  $S_1 = \{1, 2, \dots, M\}$  and  $S_2 = \{\emptyset\}$ , which means all of hops use only 1 time slot for relaying the data.

Considering the  $i$ th hop in which the data is relayed successfully with only one time slot, we can formulate the probability for this event as

$$P_i^{\text{Suc}} = \Pr[\Psi_{T_{i-1},T_i} \geq \gamma_{th}] = 1 - \text{Out}_i^{\text{DT}}. \quad (27)$$

Using (12) for (27), which yields

$$P_{i,1}^{\text{Suc}} = \frac{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},T_i}\gamma_{th} + \lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q}. \quad (28)$$

Next, if the  $i$ th hop has to use two time slots for transmitting the data, the successful probability in this case is calculated by

$$\begin{aligned} P_{i,2}^{\text{Suc}} &= \Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}, \Psi_{R_i,T_i} \geq \gamma_{th}] \\ &= \Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}] \\ &\quad \times (1 - \Pr[\Psi_{R_i,T_i} < \gamma_{th}]). \end{aligned} \quad (29)$$

Substituting (A.5) and (A.6) into (29), and after some simple manipulation, we obtain

$$\begin{aligned} P_{i,2}^{\text{Suc}} &= \frac{\lambda_{T_{i-1},\text{PU}}\lambda_{T_{i-1},T_i}\gamma_{th}(1 - \kappa\gamma_{th})Q}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + \lambda_{T_{i-1},R_i}\gamma_{th}} \\ &\quad \times \frac{1}{\lambda_{T_{i-1},\text{PU}}(1 - \kappa\gamma_{th})Q + (\lambda_{T_{i-1},T_i} + \lambda_{T_{i-1},R_i})\gamma_{th}} \\ &\quad \times \frac{\lambda_{R_i,\text{PU}}(1 - \kappa\gamma_{th})Q}{\lambda_{R_i,T_i}\gamma_{th} + \lambda_{R_i,\text{PU}}(1 - \kappa\gamma_{th})Q}. \end{aligned} \quad (30)$$

Moreover, the average number of time slots per a successful transmission in the IR protocol can be

$$P_{\text{out}}^{\text{CC}} \stackrel{Q \rightarrow +\infty}{\approx} \sum_{i=1}^M \left[ \frac{2\lambda_{T_{i-1},R_i}\lambda_{T_{i-1},T_i}}{(\lambda_{T_{i-1},\text{PU}})^2} \left( \frac{\gamma_{th}}{1 - \kappa\gamma_{th}} \right)^2 + \frac{\lambda_{T_{i-1},T_i}\lambda_{R_i,T_i}}{\lambda_{T_{i-1},\text{PU}}\lambda_{R_i,\text{PU}}} \left( -\frac{\gamma_{th}}{\kappa(2 - \kappa\gamma_{th})} - \frac{2 \log(1 - \kappa\gamma_{th})}{\kappa^2(2 - \kappa\gamma_{th})^2} \right) \right] \frac{1}{Q^2}. \quad (25)$$

formulated as follows:

$$\bar{N} = \frac{\sum_{S_1, S_2} (L + 2(M - L)) \prod_{i=1}^L P_{j_i,1}^{\text{Suc}} \prod_{i=L+1}^M P_{j_i,2}^{\text{Suc}}}{1 - P_{\text{out}}^{\text{IR}}}, \quad (31)$$

where  $1 - P_{\text{out}}^{\text{IR}}$  is the total probability that the transmission from the source to the destination is successful and  $L + 2(M - L)$  is the number of time slots used in each case of  $S_1$  and  $S_2$ .

Substituting (28), (30) into (31), we obtain an exact expression for the average number of time slots in the IR protocol.

#### 4. Simulation Results

In this section, we present Monte Carlo simulation results to verify the theoretical results and to compare the outage performance of the protocols discussed in the previous sections. In simulation environment, we consider a two-dimensional plane in which the co-ordinates of nodes  $T_i$ ,  $R_{i+1}$  and PU are  $(i/M, 0)$ ,  $((2i + 1)/2/M, 0)$  and  $(x_p, y_p)$ , respectively, where  $i \in \{0, 1, \dots, M\}$ . Therefore, the link distances can be calculated by:  $d_{T_i, T_{i+1}} = 1/M$ ,  $d_{T_i, R_{i+1}} = d_{R_{i+1}, T_{i+1}} = 1/2/M$ ,  $d_{T_i, \text{PU}} = \sqrt{(i/M - x_p)^2 + y_p^2}$  and  $d_{R_{i+1}, \text{PU}} = \sqrt{((2i + 1)/2/M - x_p)^2 + y_p^2}$ . The path-loss exponent is fixed by 4, i.e.,  $\beta = 4$ .

In Fig. 3, we present the outage probability of the DT, IR and CC protocols as a function of  $Q$  in dB. In this figure, the number of hop is fixed by 3 ( $M = 3$ ), the hardware impairment level is set to 0.1 ( $\kappa = 0.1$ ) and the outage threshold equals 0.75 ( $\gamma_{th} = 0.75$ ). In addition, we place the primary user (PU) at two different positions such as (0.3, 0.3) and (0.45, 0.45). We can observe from Fig. 3 that the IR and CC protocols obtain better performance than the DT protocol. It is because they use cooperative communication

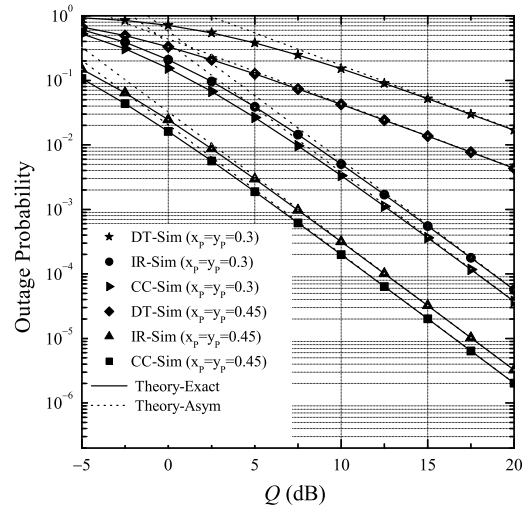


Fig. 3: Outage probability as a function of  $Q$  in dB when  $M = 3$ ,  $\kappa = 0.1$  and  $\gamma_{th} = 0.75$ .

technique at each hop, which provides higher diversity gain. As presented in this figure, the IR and CC protocols obtain the diversity order of 2, while that of the DT protocol is 1. It is also seen that the outage performance of the considered protocols significantly enhance when the PU is far the secondary network ( $x_p, y_p$  increases). Finally, it is worthy noting that the simulation results (Sim) match very well with the exactly theoretical results (Theory-Exact) and converge to the asymptotically theoretical results (Theory-Asym) at high  $Q$  region, which validates our derivations.

Figure 4 illustrates the outage probability as a function of  $\kappa$  with different values of  $\gamma_{th}$ , i.e.,  $\gamma_{th} = 1, 2$ . The remaining parameters are fixed by  $M = 4$ ,  $Q = 0$  dB,  $x_p = 0.3$  and  $y_p = 0.4$ . It can be observed from Fig. 4 that the outage probability of the DT, IR and CC protocols increases with the increasing of  $\kappa$ . Also, the CC protocol obtains the best performance because the MRC technique is



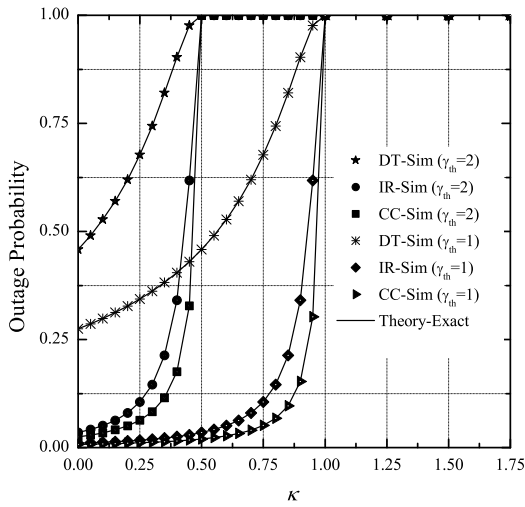


Fig. 4: Outage probability as a function of  $\kappa$  when  $M = 4$ ,  $Q = 0\text{dB}$ ,  $x_P = 0.3$  and  $y_P = 0.4$ .

equipped at the receivers. Moreover, as we can see, once  $\kappa$  is larger than  $1/\gamma_{th}$ , all of the schemes are always in outage.

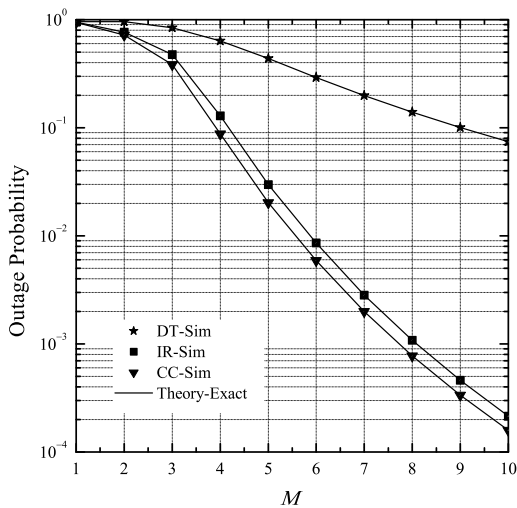


Fig. 5: Outage probability as a function of  $M$  when  $Q = 0\text{dB}$ ,  $x_P = 0.4$ ,  $y_P = 0.3$ ,  $\kappa = 0.1$  and  $\gamma_{th} = 1.25$ .

In Fig. 5, we present the outage performance as a function of the number of hops  $M$  when  $Q = 0\text{dB}$ ,  $x_P = 0.4$ ,  $y_P = 0.3$ ,  $\kappa = 0.1$  and

$\gamma_{th} = 1.25$ . We can see from this figure that the outage probability of the considered protocols decreases when the number of hops increases. It is due to the fact that with high number of hops, the distance between two intermediate nodes at each hop decreases and hence the communication between them is more reliable. However, we should note that when increasing the number of hops, the delay time from end to end also increases.

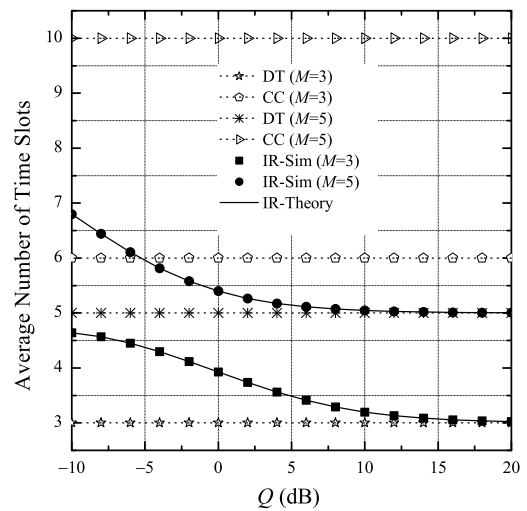


Fig. 6: Average number of time slots as a function of  $Q$  in dB when  $\kappa = 0.08$ ,  $\gamma_{th} = 1$  and  $x_P = y_P = 0.3$ .

In Fig. 6, the average number of time slots per a successful data transmission between the source and the destination is presented as a function of  $Q$  in dB. The parameters in this figure are set by  $\kappa = 0.08$ ,  $\gamma_{th} = 1$  and  $x_P = y_P = 0.3$ . As mentioned above, the DT and CC protocols use  $M$  and  $2M$  time slots to transmit the data, respectively, while, as observed from Fig. 6, the average number of time slots used in the IR protocol is between that of the DT and CC protocols. Furthermore, at high  $Q$  values, the time slots used in the IR protocol coverages to that in the DT protocol. It is because at high  $Q$  values, each hop only uses 1 time slot to relay the data.

In the last figure (Fig. 7), we compare the performance of the DT, IR and CC protocols in

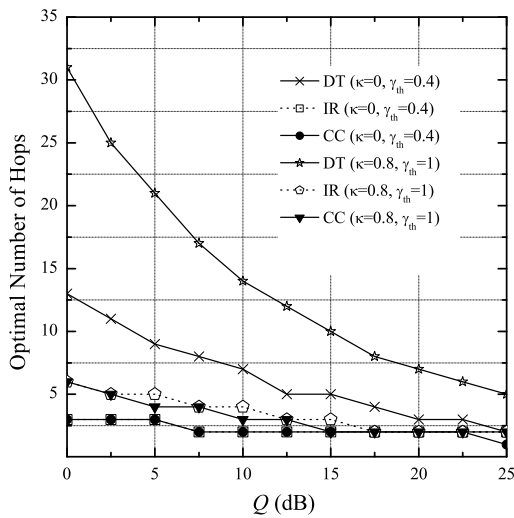


Fig. 7: Minimum number of hops as a function of  $Q$  in dB when  $x_p = 0.5$ ,  $y_p = 0.6$  and  $\varepsilon = 10^{-3}$ .

terms of optimal number of hops that is defined as minimum number of hops at which the outage probability of the X protocol is lower than a pre-determined value  $\varepsilon$ , i.e.,  $P_{\text{out}}^X \leq \varepsilon$ ,  $X \in \{\text{DT}, \text{IR}, \text{CC}\}$ . As we can observe from Fig. 7, when  $Q = 5$  dB,  $\kappa = 0.8$  and  $\gamma_{th} = 1$ , in order to satisfy the QoS, i.e.,  $P_{\text{out}}^X \leq 10^{-3}$ , the DT, IR and CC protocols need at least 21, 5 and 4 hops, respectively. It is also observed that the optimal number of hops of the DT protocol is very higher than that of the IR and CC protocols at low  $Q$  region, while that of the IR and CC protocols is almost same.

## 5. Conclusions

In this paper, we evaluated outage performance of multi-hop protocols in underlay cognitive radio networks under the impact of hardware noises. In particular, we derived the closed-form expressions of outage probability and average number of time slots over Rayleigh fading channels. Monte-Carlo simulations were then performed to verify the derivations. The interesting results in this paper can be summarized as follows:

- Under the impact of imperfect transceiver hardware, the cooperative-based multi-hop protocols still obtain the diversity order of 2. However, they are suffered from the coding gain loss due to the hardware impairment level. Finally, if the impairment level is larger than one over the outage threshold, all of the considered protocols are always in outage.
- With the same number of hops, the conventional cooperative (CC) protocol uses twice as many time slots as the direct transmission (DT) protocol, while the time slots used in the incremental cooperative protocol (IR) is between that of the CC and DT protocols. Moreover, at high  $Q$  values, that of the DT and IR protocols is same.
- To satisfy a predetermined QoS, the DT protocol uses more number of hops than the IR and CC. Moreover, the optimal number of hops used the IR and CC protocols is almost same.

## Acknowledgement

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## Appendix A: Proof of Proposition 1

At first, we consider the first term  $\mathcal{I}_1 = \Pr[\Psi_{T_{i-1}, T_i} < \gamma_{th}, \Psi_{T_{i-1}, R_i} < \gamma_{th}]$  in (17). Under the condition  $\kappa < 1/\gamma_{th}$ , by substituting  $\Psi_{T_{i-1}, T_i}$  and  $\Psi_{T_{i-1}, R_i}$  in (8) into  $\mathcal{I}_1$ , we have

$$\mathcal{I}_1 = \Pr \left[ \frac{\gamma_{T_{i-1}, T_i}}{\gamma_{T_{i-1}, \text{PU}}} < \rho, \frac{\gamma_{T_{i-1}, R_i}}{\gamma_{T_{i-1}, \text{PU}}} < \rho \right], \quad (\text{A.1})$$

where  $\rho = \gamma_{th} / (1 - \kappa\gamma_{th})$ .

From (A.1), we can rewrite  $\mathcal{I}_1$  under the following form:

$$\mathcal{I}_1 = \int_0^{+\infty} F_{\gamma_{T_{i-1}, T_i}}(\rho x) F_{\gamma_{T_{i-1}, R_i}}(\rho x) f_{\gamma_{T_{i-1}, \text{PU}}}(x) dx. \quad (\text{A.2})$$

Substituting the CDFs  $F_{\gamma_{T_{i-1},T_i}}(\rho x)$ ,  $F_{\gamma_{T_{i-1},R_i}}(\rho x)$  and the PDF  $f_{\gamma_{T_{i-1},PU}}(x)$  obtained from (1) into (A.2), and after some manipulation, we can obtain

$$\mathcal{I}_1 = 1 - \frac{\lambda_{T_{i-1},PU}}{\lambda_{T_{i-1},PU} + \lambda_{T_{i-1},T_i}\rho} - \frac{\lambda_{T_{i-1},PU}}{\lambda_{T_{i-1},PU} + \lambda_{T_{i-1},R_i}\rho} + \frac{\lambda_{T_{i-1},PU}}{\lambda_{T_{i-1},PU} + (\lambda_{T_{i-1},T_i} + \lambda_{T_{i-1},R_i})\rho}. \quad (\text{A.3})$$

Next, for the secondary term  $\mathcal{I}_2 = \Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}, \Psi_{R_i,T_i} < \gamma_{th}]$  in (17), it can be rewritten as

$$\mathcal{I}_2 = \Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}] \times \Pr[\Psi_{R_i,T_i} < \gamma_{th}]. \quad (\text{A.4})$$

In the above equation,  $\Pr[\Psi_{R_i,T_i} < \gamma_{th}]$  can be obtained similarly as (12)

$$\Pr[\Psi_{R_i,T_i} < \gamma_{th}] = \frac{\lambda_{R_i,T_i}\gamma_{th}}{\lambda_{R_i,T_i}\gamma_{th} + \lambda_{R_i,PU}(1 - \kappa\gamma_{th})Q}. \quad (\text{A.5})$$

Moreover, with the same manner, we can calculate the probability  $\Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}]$  in (A.4) by

$$\begin{aligned} & \Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}] = \\ & \int_0^{+\infty} F_{\gamma_{T_{i-1},T_i}}(\rho x)(1 - F_{\gamma_{T_{i-1},R_i}}(\rho x))f_{\gamma_{T_{i-1},PU}}(\rho x)dx \\ & = \frac{\lambda_{T_{i-1},PU}}{\lambda_{T_{i-1},PU} + \lambda_{T_{i-1},R_i}\rho} \\ & - \frac{\lambda_{T_{i-1},PU}}{\lambda_{T_{i-1},PU} + (\lambda_{T_{i-1},T_i} + \lambda_{T_{i-1},R_i})\rho}. \end{aligned} \quad (\text{A.6})$$

Combining (17), (A.3), (A.4), (A.5) and (A.6) and  $\rho = \gamma_{th}/(1 - \kappa\gamma_{th})$  together, we obtain (18).

### Appendix B: Proof of Corollary 1

At high Q values, i.e.,  $Q \rightarrow +\infty$  or  $\rho \rightarrow 0$ , by applying the approximation:  $1 - \exp(-x) \stackrel{x \rightarrow 0}{\approx} x$  for the CDFs in (A.2), we have

$$F_{\gamma_{T_{i-1},T_i}}(\rho x) \stackrel{Q \rightarrow +\infty}{\rho \rightarrow 0} \approx \lambda_{T_{i-1},T_i}\rho x, \quad (\text{B.1})$$

$$F_{\gamma_{T_{i-1},R_i}}(\rho x) \stackrel{Q \rightarrow +\infty}{\rho \rightarrow 0} \approx \lambda_{T_{i-1},R_i}\rho x. \quad (\text{B.2})$$

Plugging (B.1) and (A.2) together yields

$$\begin{aligned} \mathcal{I}_1 & \stackrel{Q \rightarrow +\infty}{\approx} \lambda_{T_{i-1},R_i}\lambda_{T_{i-1},T_i}\rho^2 \\ & \times \int_0^{+\infty} x^2 \lambda_{T_{i-1},PU} \exp(-\lambda_{T_{i-1},PU}x) dx \\ & \stackrel{Q \rightarrow +\infty}{\approx} \frac{2\lambda_{T_{i-1},R_i}\lambda_{T_{i-1},T_i}}{(\lambda_{T_{i-1},PU})^2} \rho^2. \end{aligned} \quad (\text{B.3})$$

With the same manner, we also obtain

$$\Pr[\Psi_{T_{i-1},T_i} < \gamma_{th}, \Psi_{T_{i-1},R_i} \geq \gamma_{th}] \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{T_{i-1},T_i}}{\lambda_{T_{i-1},PU}} \rho. \quad (\text{B.4})$$

Moreover, by using (14), we also have

$$\Pr[\Psi_{R_i,T_i} < \gamma_{th}] \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{R_i,T_i}}{\lambda_{R_i,PU}} \rho. \quad (\text{B.5})$$

Substituting (B.3)-(B.5) and  $\rho = \gamma_{th}/(1 - \kappa\gamma_{th})$  into (17), we can obtain (20) and finish the proof here.

### Appendix C: Proof of Proposition 2

In this Appendix, we only focus on deriving the outage probability  $\mathcal{I}_3 = \Pr[\Psi_{T_{i-1},R_i} \geq \gamma_{th}, \Psi_{MRC} < \gamma_{th}]$ . At first, we should note that RVs  $\Psi_{T_{i-1},R_i}$  and  $\Psi_{MRC}$  are not independent because they include a common RV, i.e.,  $\gamma_{T_{i-1},PU}$ . Similar to [32, 33], by considering  $\gamma_{T_{i-1},PU}$  as a constant, i.e., setting  $\gamma_{T_{i-1},PU} = x$ , we can obtain a conditioned probability for  $\mathcal{I}_3$  as follows:

$$\begin{aligned} \mathcal{I}_3(x) & = \Pr[\Psi_{T_{i-1},R_i} \geq \gamma_{th}, \Psi_{MRC} < \gamma_{th} | \gamma_{T_{i-1},PU} = x] \\ & = \Pr\left[\frac{Q\gamma_{T_{i-1},R_i}/x}{\kappa Q\gamma_{T_{i-1},R_i}/x + 1} \geq \gamma_{th}\right] \times \\ & \underbrace{\Pr\left[\frac{Q\gamma_{T_{i-1},T_i}/x}{\kappa Q\gamma_{T_{i-1},T_i}/x + 1} + \frac{Q\gamma_{R_i,T_i}/\gamma_{R_i,PU}}{\kappa Q\gamma_{R_i,T_i}/\gamma_{R_i,PU} + 1} < \gamma_{th}\right]}_{\mathcal{I}_4}. \end{aligned} \quad (\text{C.1})$$

In addition, it is easy to obtain that

$$\begin{aligned} & \Pr\left[\frac{Q\gamma_{T_{i-1},R_i}/x}{\kappa Q\gamma_{T_{i-1},R_i}/x + 1} \geq \gamma_{th}\right] = \Pr\left[\gamma_{T_{i-1},R_i} \geq \frac{\rho x}{Q}\right] \\ & = \exp\left(-\frac{\lambda_{T_{i-1},R_i}\rho}{Q}x\right). \end{aligned} \quad (\text{C.2})$$

Before calculating  $\mathcal{I}_4$  in (C.1), we introduce two RVs:  $Z_1 = \frac{Q\gamma_{T_{i-1},T_i}/x}{\kappa Q\gamma_{T_{i-1},T_i}/x+1}$  and  $Z_2 = \frac{Q\gamma_{R_i,T_i}/\gamma_{R_i,PU}}{\kappa Q\gamma_{R_i,T_i}/\gamma_{R_i,PU}+1}$ . First, we attempt to find the CDF of  $Z_1$ , which can be given by

$$F_{Z_1}(z) = \Pr[Z_1 < z] = \begin{cases} 1; & \text{if } z \geq 1/\kappa \\ 1 - \exp\left(-\lambda_{T_{i-1},T_i} \frac{x}{Q} \frac{z}{1-\kappa z}\right); & \text{if } z < 1/\kappa \end{cases} \quad (\text{C.3})$$

Then, the corresponding PDF  $f_{Z_1}(z)$  can be obtained from (C.3) as

$$f_{Z_1}(z) = \frac{\partial F_{Z_1}(z)}{\partial z} = \begin{cases} 0; & \text{if } z \geq 1/\kappa \\ \frac{\lambda_{T_{i-1},T_i} x}{Q(1-\kappa z)^2} \exp\left(-\lambda_{T_{i-1},T_i} \frac{x}{Q} \frac{z}{1-\kappa z}\right); & \text{if } z < 1/\kappa \end{cases} \quad (\text{C.4})$$

For the RV  $Z_2$ , by using [29, eq. (3)], we can obtain the CDF  $F_{Z_2}(t)$  as

$$F_{Z_2}(t) = \Pr[Z_2 < t] = \begin{cases} 1; & \text{if } t \geq 1/\kappa \\ \frac{\lambda_{R_i,T_i} t}{\lambda_{R_i,PU} Q(1-\kappa t) + \lambda_{R_i,T_i} t}; & \text{if } t < 1/\kappa \end{cases} \quad (\text{C.5})$$

With the results obtained in (C.3)-(C.5), the probability  $\mathcal{I}_4$  in (C.1) can be expressed by the following formula:

$$\begin{aligned} \mathcal{I}_4 &= \Pr[Z_1 + Z_2 < \gamma_{th}] \\ &= \int_0^{\gamma_{th}} f_{Z_1}(z) F_{Z_2}(\gamma_{th} - z) dz \\ &= \int_0^{\gamma_{th}} \frac{\lambda_{T_{i-1},T_i} x}{Q(1-\kappa z)^2} \exp\left(-\lambda_{T_{i-1},T_i} \frac{x}{Q} \frac{z}{1-\kappa z}\right) \\ &\quad \times \frac{\lambda_{R_i,T_i} (\gamma_{th} - z)}{\lambda_{R_i,PU} Q(1-\kappa(\gamma_{th} - z)) + \lambda_{R_i,T_i} (\gamma_{th} - z)} dz. \end{aligned} \quad (\text{C.6})$$

Moreover,  $\mathcal{I}_3$  can be obtained from  $\mathcal{I}_3(x)$  by using the following rule:

$$\mathcal{I}_3 = \int_0^{+\infty} \mathcal{I}_3(x) f_{\gamma_{T_{i-1},PU}}(x) dx. \quad (\text{C.7})$$

Plugging (C.1), (C.2), (C.6) and (C.7) together, and after some manipulation, we can get (C.8) as

shown at the top of next page.

Let us consider  $\mathcal{I}_5$  in (C.8), it can be rewritten under the following form

$$\begin{aligned} \mathcal{I}_5 &= \frac{a_0 (\gamma_{th} - z)}{(a_1 - z)^2 (a_2 - z)} \\ &= a_0 \left( \frac{b_1}{(a_1 - z)^2} + \frac{b_2}{a_1 - z} - \frac{b_2}{a_2 - z} \right), \end{aligned} \quad (\text{C.9})$$

where,

$$\begin{aligned} a_0 &= \frac{\lambda_{T_{i-1},T_i} \lambda_{R_i,T_i} \lambda_{T_{i-1},PU} Q}{\left( \kappa (\lambda_{T_{i-1},PU} Q + \lambda_{T_{i-1},R_i} \rho) - \lambda_{T_{i-1},T_i} \right)^2} \\ &\quad \times \frac{1}{(\lambda_{R_i,T_i} - \kappa \lambda_{R_i,PU} Q)}, \\ a_1 &= \frac{\lambda_{T_{i-1},PU} Q + \lambda_{T_{i-1},R_i} \rho}{\kappa (\lambda_{T_{i-1},PU} Q + \lambda_{T_{i-1},R_i} \rho) - \lambda_{T_{i-1},T_i}}, \\ a_2 &= \frac{\lambda_{R_i,PU} Q (1 - \kappa \gamma_{th}) + \lambda_{R_i,T_i} \gamma_{th}}{\lambda_{R_i,T_i} - \kappa \lambda_{R_i,PU} Q}, \\ b_1 &= \frac{\gamma_{th} - a_1}{a_2 - a_1}, \quad b_2 = -\frac{\gamma_{th} - a_2}{(a_1 - a_2)^2}. \end{aligned} \quad (\text{C.10})$$

It is noted from (C.9) that for ease of presentation and analysis, we assume that  $a_1 \neq a_2$ . Next, substituting (C.9) into (C.8), we obtain

$$\mathcal{I}_3 = a_0 \left( \frac{b_1 \gamma_{th}}{a_1 (a_1 - \gamma_{th})} + b_2 \log \left( \frac{a_1 (a_2 - \gamma_{th})}{(a_1 - \gamma_{th}) a_2} \right) \right). \quad (\text{C.11})$$

Finally, substituting (A.3) and (C.11) into (22), we easily obtain (23).

#### Appendix D: Proof of Corollary 2

This appendix focus on deriving the asymptotic expression for  $\mathcal{I}_3 = \Pr[\Psi_{T_{i-1},R_i} \geq \gamma_{th}, \Psi_{MRC} < \gamma_{th}]$  at high  $Q$  value. At first, from (C.2), we can obtain

$$\Pr \left[ \frac{Q\gamma_{T_{i-1},R_i}/x}{\kappa Q\gamma_{T_{i-1},R_i}/x+1} \geq \gamma_{th} \right] \stackrel{Q \rightarrow +\infty}{\approx} 1. \quad (\text{D.1})$$

Secondly, when  $t < 1/\kappa$ , we can approximate  $f_{Z_1}(z)$  as follows:

$$f_{Z_1}(z) \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{T_{i-1},R_i} x}{Q(1-\kappa z)^2}. \quad (\text{D.2})$$

$$I_3 = \int_0^{\gamma_{th}} \frac{Q\lambda_{T_{i-1},T_i}\lambda_{T_{i-1},PU}}{\underbrace{[(\lambda_{T_{i-1},PU}Q + \lambda_{T_{i-1},R_i\rho})(1 - \kappa z) + \lambda_{T_{i-1},T_i}z]^2}_{I_5}} \frac{\lambda_{R_i,T_i}(\gamma_{th} - z)}{\lambda_{R_i,PU}Q(1 - \kappa(\gamma_{th} - z)) + \lambda_{R_i,T_i}(\gamma_{th} - z)} dz. \quad (C.8)$$

Moreover, with  $t < 1/\kappa$ , applying the approximation in (14) for (C.5) yields

$$F_{Z_2}(t) \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{R_i,T_i}}{\lambda_{R_i,PU}} \frac{t}{1 - \kappa t} \frac{1}{Q}. \quad (D.3)$$

With the same manner as in Appendix C and with the approximation obtained in (D.1), (D.2) and (D.3), we can approximate  $I_3$  as follows:

$$I_3 \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{T_{i-1},T_i}\lambda_{R_i,T_i}}{\lambda_{T_{i-1},PU}\lambda_{R_i,PU}} \frac{1}{Q^2} \times \int_0^{\gamma_{th}} \underbrace{\frac{1}{(1 - \kappa z)^2} \frac{\gamma_{th} - z}{1 - \kappa\gamma_{th} + \kappa z}}_{I_6} dz. \quad (D.4)$$

To calculate the integral in (D.4), we can rewrite  $I_6$  under the following form:

$$I_6 = \frac{\kappa\gamma_{th} - 1}{\kappa(2 - \kappa\gamma_{th})} \frac{1}{(1 - \kappa z)^2} + \frac{1}{\kappa(2 - \kappa\gamma_{th})^2} \frac{1}{1 - \kappa z} + \frac{1}{\kappa(2 - \kappa\gamma_{th})^2} \frac{1}{1 - \kappa\gamma_{th} + \kappa z}. \quad (D.5)$$

Substituting (D.5) into (D.4), we can obtain

$$I_3 \stackrel{Q \rightarrow +\infty}{\approx} \frac{\lambda_{T_{i-1},T_i}\lambda_{R_i,T_i}}{\lambda_{T_{i-1},PU}\lambda_{R_i,PU}} \times \left( -\frac{\gamma_{th}}{\kappa(2 - \kappa\gamma_{th})} - \frac{2 \log(1 - \kappa\gamma_{th})}{\kappa^2(2 - \kappa\gamma_{th})^2} \right) \frac{1}{Q^2}. \quad (D.6)$$

Substituting (B.3) and (D.6) into (22), we obtain

$$\text{Out}_i^{\text{CC}} \stackrel{Q \rightarrow +\infty}{\approx} \frac{2\lambda_{T_{i-1},R_i}\lambda_{T_{i-1},T_i}}{(\lambda_{T_{i-1},PU})^2} \left( \frac{\gamma_{th}}{1 - \kappa\gamma_{th}} \right)^2 \frac{1}{Q^2} + \frac{\lambda_{T_{i-1},T_i}\lambda_{R_i,T_i}}{\lambda_{T_{i-1},PU}\lambda_{R_i,PU}} \left( -\frac{\gamma_{th}}{\kappa(2 - \kappa\gamma_{th})} - \frac{2 \log(1 - \kappa\gamma_{th})}{\kappa^2(2 - \kappa\gamma_{th})^2} \right) \frac{1}{Q^2}. \quad (D.7)$$

Using (D.7) for (24), we can obtain (25).

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